A Statistical Equilibrium Approach to Adam Smith's Labor Theory of Value

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Introduction

- Investigation of Adam Smith's ideas about self-organization of society into a **social division of labor** from a **statistical equilibrium** perspective.
- A large number of **independent decentralized** producers who are free to decide **what to produce** might **organize the social division of labor** to meet the needs of social reproduction and unlock the social benefits of capitalism (economic growth)
- Free competition $\rightarrow$ division of labor $\rightarrow$ increases the extent of the market $\rightarrow$ increases labor productivity

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Motivation

- **Social coordination problem:** organizing a social division of labor.
- This problem is **statistical** and focuses on the equilibrium conditions of the endogenous distribution of producers in the long-run (stability).
- **Problem:** producers seek to maximize their individual rates of return on production.
  - **Primal:** fluctuations of individual producers in and out of different lines of production.
  - **Dual:** fluctuations of market prices around natural prices.

Motivation

- Smith's LTV: **two feedbacks**
  1. $P(action \mid payoff)$: Independent producers **migrate** from low-payoff sectors to higher-payoff sectors, with payoff measured as their income relative to labor effort.
  2. $P(payoff \mid action)$: The movement of producers **into** (out of) a sector tends to **lower** (raise) the payoff through the dual movement of prices.
- $1 + 2 \rightarrow$ **convergence** of market to natural prices.
- **Markov process** describing the **stochastic movement of producers** with an **ergodic distribution** that **on average** "*balances the advantages and disadvantages*" of production and implies **market prices** will **gravitate** around **natural prices**.

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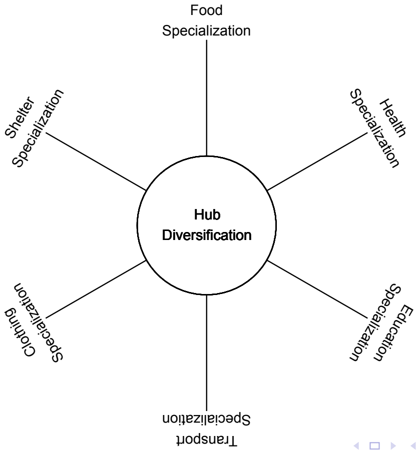
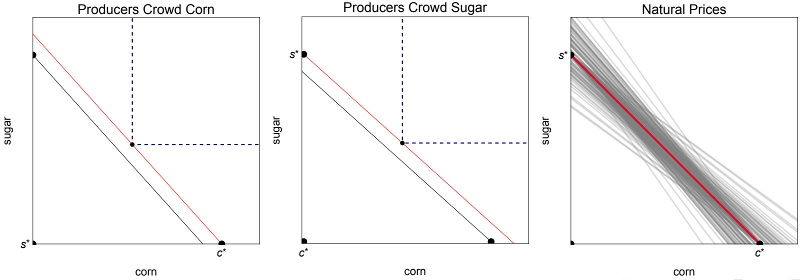
Hub-and-spoke model

- $N >> 1$ identical independent producers and K spokes, each representing a necessary line of production.
- n_k is the number of producers in spoke k, and $\sum_{k=1}^{K} n_k = N$ is the total number of producers, defining degrees of freedom of the system.



Specialized production

- Producer's decision: **diversify** or **specialize**?
- It depends on: **(i)** the shape of the feasible frontier, **(ii)** the relative prices of the goods, and **(iii)** on the context the producers find themselves.
- With continuous **migration of labor** over and undershooting the relative advantage of employment, a center of gravity emerges around competitive **natural prices**: at natural prices commodities will exchange at their values reflecting embodied labor time, $p \propto \frac{\lambda_c}{\lambda_s}$.



Division of labor with two perishable goods

- Total quantity of corn (X) and sugar (Y) produced is equivalent to the number of producers in each respective category $\rightarrow n_c = \frac{X}{N}$ and $n_s = \frac{Y}{N} = 1 - n_c$

- **"Short-side" power** (Bowles [2022]): In markets that don't clear, the short-side (excess demand) has power over long-side (excess supply) due to fall in prices.
- In the hub-and-spoke model an excess supply of corn implies $n_c > \frac{1}{2}$ in which case corn will be on the "long side" and sugar on the "short side".

Division of labor with two perishable goods

- Typical producer with a Leontieff payoff function:
- Sugar producers are on the "short-side" and corn producers are on the "long-side" of the market

$$n_c > \frac{1}{2} \begin{cases} \text{With Prob.} = 1 - n_c \begin{cases} \text{Corn: } \frac{X}{N_s} = \frac{n_c}{1-n_c} \\ \text{Sugar: } \frac{Y}{N_s} = 1 \end{cases} & \rightarrow \min\left[\frac{n_c}{1-n_c}, 1\right] = 1 \\ \text{With Prob.} = n_c \begin{cases} \text{Corn: } 0 \\ \text{Sugar: } 0 \end{cases} & \rightarrow \min[0, 0] = 0 \end{cases} \quad (1)$$

- Corn producers are on the "short-side" and sugar producers are on the "long-side" of the market

$$n_c < \frac{1}{2} \begin{cases} \text{With Prob.} = n_c \begin{cases} \text{Corn: } \frac{X}{N_c} = 1 \\ \text{Sugar: } \frac{Y}{N_c} = \frac{n_s}{1-n_s} = \frac{1-n_c}{n_c} \end{cases} & \rightarrow \min\left[1, \frac{1-n_c}{n_c}\right] = 1 \\ \text{With Prob.} = 1 - n_c \begin{cases} \text{Corn: } 0 \\ \text{Sugar: } 0 \end{cases} & \rightarrow \min[0, 0] = 0 \end{cases} \quad (2)$$

Division of labor with two perishable goods

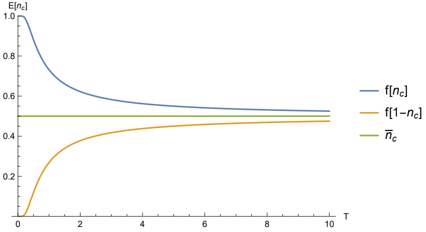
- A simple and parsimonious way of modeling the partial **randomization of strategies** is by constraining the typical producer's mixed strategy with a **minimum informational entropy**. Foley [2020]; Scharfenaker [2020]
- The solution to the constrained maximization problem is the Gibbs (SoftMax) distribution over actions

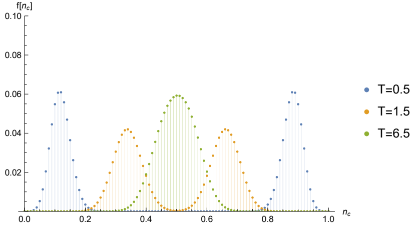
$$f\left(n_c(t+1)|n_c(t)\right) = \begin{cases} \frac{1}{1+e^{\frac{Min[0,0]-Min\left[1,\frac{1-n_c(t)}{n_c(t)}\right]}{T}}} = \frac{1}{1+e^{-\frac{1}{T}}} & \text{if } n_c(t) < \frac{1}{2} \\ \frac{1}{1+e^{-\frac{Min\left[1,\frac{1-n_c(t)}{n_c(t)}\right]-Min(0,0)}{T}}} = \frac{1}{1+e^{\frac{1}{T}}} & \text{if } n_c(t) > \frac{1}{2} \end{cases} \quad (3)$$

Division of labor with two perishable goods

- The stochastic quantal response of the typical producer induces a **Markov chain on the state space of profiles of agent behavior**.
- If there are N producers each with the same behavioral temperature T, the state of the system (**distribution of producers**) is described by the number of producers choosing corn, $N_c = 0, 1, ..., N$. The frequency with which each producer will choose corn is $f(n_c) = \frac{1}{1+e^{-\frac{1}{T}}}$ and takes the **Binomial** form:

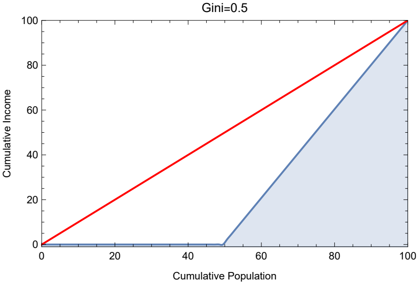
$$\binom{N}{1-N_c} f(n_c)^{1-N_c}(1-f(n_c))^{N-(1-N_c)} \tag{4}$$





Endogenous inequality

- The **blue line** is the Lorenz curve corresponding to the cumulative percentage of the population and the cumulative percentage of goods owned by that portion of the population.
- The **red line** is the line of perfect equality, which would represent a situation where each person has the same share of goods.
- The Gini coefficient, ratio of the area between the Lorenz cuve and 45−degree line to the total area under the 45−degree line is 0.5.



Production with durable good

- Each producer has an individual **steel stock** $y_i(t) \in [0, \bar{Y}(t)]$ before consumption in any period.
- **Corn** is on the "**long-side**" of the market when the **newly produced corn** is greater than **total** amount of **steel** on the market

$$
\begin{aligned}
n_c(t) &> (1 - n_c(t)) + \bar{y}_c(t) + \bar{y}_s(t) \\
2n_c(t) &> \bar{y}_c(t) + \bar{y}_s(t) + 1 \\
n_c(t) &> \frac{\bar{y}(t) + 1}{2}
\end{aligned}
\tag{5}
$$

- When commodities are shared among producers we hold an hypothesis of **equal distribution**.

Production with durable good

Steel accumulation when corn is on the "long-side" and steel on the "short-side"

- **Corn producers - corn "long"**
  - Corn weak: they consume their production of corn (accumulation possible) or their steel stock (no accumulation), whichever is lower.

- **Steel producers - steel "short"**
  - Steel strong: consume either corn or steel stock. Depends how much corn is available.
    - If there is no excess corn, they can consume 0 corn or their stock. As 0 is less than a positive stock, there is no accumulation because steel stock in $t+1=t$
    - If there is excess corn, it is equally distributed among steel producers. They choose between excess corn or their steel (production + stock). Accumulation is possible.

Production with durable good

Steel accumulation when corn is on the "short side" and steel on the "long side"

- Steel producers don't own a corn stock.
  - Corn producers get: all corn produced + steel stock from both producers
- **Corn producers - corn "short"**
  - Corn strong: they consume their production of corn (accumulation possible) or all steel stock available (no accumulation), whichever is lower.
- **Steel producers - steel "long"**
  - Steel weak: They get nothing. No accumulation occurs.

Summary

Payoff Matrix

	Corn Short $(n_c[t] < (1 - n_c[t]) + \bar{y}[t])$	Steel Short $(n_c[t] > (1 - n_c[t]) + \bar{y}[t])$
Corn Producer	$\text{Min}\left[1, y_i^c[t] + \frac{(1-n_c[t])+\bar{y}^S}{n_c[t]}\right] = \begin{cases} 1 & \text{if } y_i^c[t] + \frac{(1-n_c[t])+\bar{y}^S}{n_c[t]} > 1 \\ y_i^c[t] + \frac{(1-n_c[t])+\bar{y}^S}{n_c[t]} & \text{otherwise} \end{cases}$	$\text{Min}[1, y_i^c[t]] = \begin{cases} 1 & \text{if } y_i^c[t] > 1 \\ y_i^c[t] & \text{otherwise} \end{cases}$
Steel Producer	$\text{Min}[0, 0] = 0$	$\begin{cases} \text{Min}[0, y_i^s[t]] = 0 & \text{if } y_i^c[t] > 1\ \forall\, i \\ \text{Min}\left[\frac{n_c[t]-\bar{y}^C[t]}{1-n_c[t]}, y_i^s[t] + 1\right] = \begin{cases} y_i^s[t] + 1 & \text{if } \frac{n_c[t]-\bar{y}^C[t]}{1-n_c[t]} - y_i^s[t] > 1 \\ \frac{n_c[t]-\bar{y}^C[t]}{1-n_c[t]} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$

Accumulation Matrix

	Corn Short $(n_c[t] < (1 - n_c[t]) + \bar{y}[t])$	Steel Short $(n_c[t] > (1 - n_c[t]) + \bar{y}[t])$
Corn Producer	$y_i[t+1] = \begin{cases} y_{i\in c}[t] + \left(\frac{(1-n_c[t])+\bar{y}_s[t]}{n_c[t]}\right) - 1 & \text{if } y_{i\in c}[t] + \left(\frac{(1-n_c[t])+\bar{y}_s[t]}{n_c[t]}\right) > 1 \\ 0 & \text{otherwise} \end{cases}$	$y_i^c[t+1] = \begin{cases} y_i^c[t] - 1 & \text{if } y_i^c[t] > 1 \\ 0 & \text{otherwise} \end{cases}$
Steel Producer	$y_{i\in s}[t+1] = 0$	$\begin{cases} y_{i\in s}[t+1] = y_{i\in s}[t] & \text{if } y_{i\in c}[t] > 1\ \forall\, i \\ y_i^s[t+1] = \begin{cases} y_{i\in s}[t] + 1 - \left(\frac{n_c[t]-(\bar{y}[t]-\bar{y}_c[t])}{1-n_c[t]}\right) & \text{if } y_i^s[t] + 1 - \left(\frac{n_x[t]-(\bar{y}[t]-\bar{y}_s[t])}{1-n_c[t]}\right) > 0 \\ 0 & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$

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Evolution of state variables

- Instead of solving jointly for the joint ergodic distribution $f(n_c, y)$ we use the fact that the **payoffs must be equal in long-run steady state equilibrium**.
- While a typical producer has a $\frac{1}{2}$ chance of being either a corn or steel producer in any time period the number of corn producers that determines the "long-" and "short-side" will follows an unbiased random walk:

$$f(N_c(t+1)) = \binom{N}{1-N_c} \left(\frac{1}{2}\right)^{1-N_c} \left(\frac{1}{2}\right)^{N-(1-N_c)} \tag{6}$$

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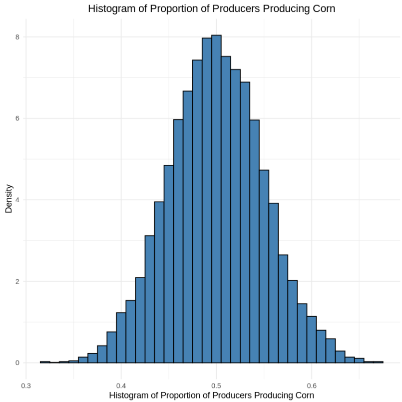
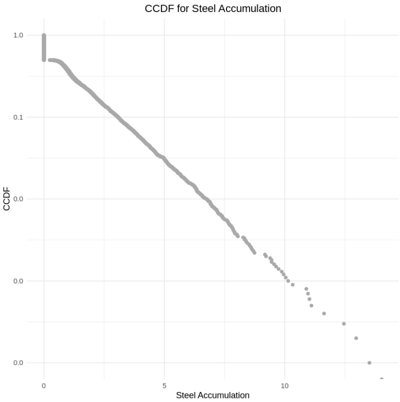
Simulation

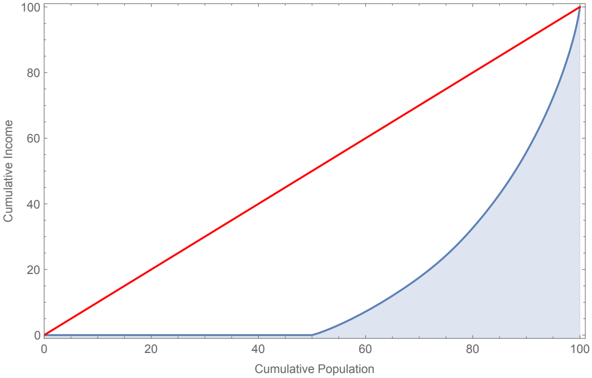




- The CCDF is plotted on a semi-logarithmic scale to emphasize the linearity of the distribution indicating an exponential distribution in the tail.

Inequality

- The Gini coefficient in the durable goods economy is 0.685.



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Conclusion

- Classical Political Economy recognized **capitalism** as a **complex social system with astronomical degrees of freedom, complex interdependencies, interactions, and numerous feedbacks**.
- Adam Smith argued that **conclusions** about capitalism must rest on robust, pervasive, self-reinforcing (**statistical**) tendencies.
- Smith's logic concerning the process of the **spontaneous formation** of the **social division of labor** and its implications for the theory of **value** is inherently **statistical**.
- **Centers of gravity** in **prices** emerge through the **endogenous fluctuations** of individual **producers** between **different lines** of **production**.
- A **statistical equilibrium** hub-and-spoke model is developed to address this irreducible **element** of **randomness** in Smith's theory.
- **Entropy-constrained independent producers** balance the "advantages and disadvantages" of employment through their stochastic **movement** between **spokes**.
- The resulting **ergodic distribution** of **producers supports** Smith's theory of **gravitational equilibrium** and the **labor theory of value**.
- The **market** introduces **endogenous inequality** in both perishable goods and durable good economy.
- **Statistical equilibrium** methods shed new light on the primary theoretical **abstractions** of Classical Political Economy by modeling the statistical processes that generate the robust predicted **regularities**.

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Loose ends

- Closed-form solution to accumulation vs. simulation.
    - The process of accumulation can be modeled with a Langevin or stochastic differential equation, such as an Itô processes driven by Brownian motion.
    - Fokker-Plank equation for the density $f(y,t)$
- Different forms of distribution (proportional vs. equal)
- From the commodity law of exchange to the capitalist law of exchange
    - No relations of production to classes and private property of the means of production.
- Discussion section
    - Long-period method
    - Disequilibrium vs. statistical equilibrium

References

Samuel Bowles. *Microeconomics: Competition, Conflict, and Coordination*. Oxford University Press, Oxford ; New York, NY, 2022. ISBN 978-0-19-884320-7.

Duncan K. Foley. Information theory and behavior. *The European Physical Journal Special Topics*, 229(9):1591–1602, July 2020. ISSN 1951-6355, 1951-6401. doi: 10.1140/epjst/e2020-900133-x.

Ellis Scharfenaker. Implications of quantal response statistical equilibrium. *Journal of Economic Dynamics and Control*, 119:103990, October 2020. ISSN 01651889. doi: 10.1016/j.jedc.2020.103990.