A Statistical Equilibrium Approach to Adam Smith's Labor Theory of Value

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Abstract

Adam Smith's inquiry into the emergence and stability of the self-organization of the division of labor in commodity exchange is considered using statistical equilibrium methods from statistical physics. We develop a statistical equilibrium model of the distribution of independent producers and produced commodities in a hub-and-spoke framework that predicts the both the center of gravity of producers across lines of production as well as the endogenous fluctuations between lines of production that arise from the institutional constraints of "perfect liberty" and free competition. The ergodic distribution of producers implies the convergence of market prices to Smith's natural prices.

Keywords: Competition, Hub-and-spoke, Value theory, Classical Political Economy, Statistical equilibrium.

JEL codes: B12, B40, B50, C18

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1 Introduction

Adam Smith's point of departure for understanding the rapidly expanding sphere of marked mediated production and exchange in 18th century England and it's attendant social benefits is an investigation of the self-organization of society into a social division of labor. His investigation of the origins of, and limits to, the division of labor introduced in the first three chapters of the Wealth of Nations [Smith, 1904] (and discussed at length in Book III), becomes the foundation for his theory of price in chapter six. Here, Smith conceptualizes his famous thought experiment in which a large number of independent decentralized producers who are free to decide what and how to produce might organize the social division of labor to meet the needs of social reproduction and unlock the social benefits of capitalism. Smith argues that the process that gives rise to the division of labor, increasing extent of the market, and increasing labor productivity arises spontaneously as an optimal outcome of decentralized decision making in the institutional setting of free competition. Smith identifies the immense social benefits of increasing labor productivity as the primary effect of the division of labor and thus the problem of realizing the "self-organization" of the division of labor in society is what occupies the first part of Book I and entirety of Book III of the Wealth of Nations.

While Smith is able to identify the positive feedbacks of the division of labor on the extent of the market once the economic circumstances that give rise to the division of labor are established, he offers a much less compelling argument as to why these circumstances came to prevail in the first place.[1] Taking the economic conditions that support an extensive division of labor as having naturally transpired, he explores the economic implications of the social division of labor in developing his theory of value and price.

Smith's thought experiment describes a social coordination problem of organizing independent specialized producers in various lines of production in order to meet the needs of social reproduction. Because producers are free to move into any line of production and because these decisions are decentralized, there will be considerable element of chance in finding any particular producer in any particular line of production at any point in time. Thus, the problem of achieving the social division of labor is inherently statistical and primarily concerns the stability and (long-run) equilibrium conditions of the distribution of individual producers [Scharfenaker and Yang, 2020]. Smith is clear in his statistical description of commodity production that the endogenous movement of producers among different lines of production results in meeting the needs of social reproduction only *on average*. In simple

[1] The problems of Neo-Smithinan historiography is well debated in the economic history literature for example, [Brenner, 2008; Hilton, 1982; Wood, 2002].

commodity production the endogenous fluctuations of payoffs for individual producers, in mathematical language of linear programming, the "dual" representation of the fluctuations of individual producers in and out of different lines of production.

There are two principal social coordination problems that Smith identifies in the first chapters of The Wealth of Nations. The first concerns the independent producers decision problem of diversifying or specializing in production. The second concerns the decision problem of independent producers in organizing a division of labor that can meet the needs of social reproduction and generate the social benefits of economic growth. Because independent producers' decisions are decentralized in both problems, Smith poses the solution to the first problem as historically determined by humans propensity to "truck, barter, and trade." Brenner in [Hilton, 1982], among others, have offered a compelling critique of the the Neo-Smithian account of capitalist development that identifies the process of capitalist expansion as naturally induced by the development of trade. While Smith devotes much of Book III discussing the historical rise of capitalism, his far more important contribution in the Wealth of Nations is his attempt to understand the essential laws and tendencies of capitalism responsible for modern economic growth that concern the second thought experiment. What Smith identifies as perhaps the most remarkable feature of capitalism is that the needs of social reproduction can be met on an ever expanding scale as "emergent" self-organized outcomes facilitated by the institutional constraints of market mediated production and exchange and the purposive behavior of producers seeking to maximize their individual rates of return on production.

Smith's theory of value and prices in its simplest form concerns two feedbacks that we can model as conditional probability distributions. The first is that independent producers who take the payoff in particular sectors of production as determined by forces beyond their control will move from sectors with relatively low payoffs to those sectors with relatively higher payoffs, where payoffs are understood as their income relative to labor effort (in Smith's language "advantages" relative to "disadvantages").

> The whole of the advantages and disadvantages of the different employments of labour and stock must, in the same neighbourhood, be either perfectly equal or continually tending to equality. If in the same neighbourhood, there was any employment evidently either more or less advantageous than the rest, so many people would crowd into it in the one case, and so many would desert it in the other, that its advantages would soon return to the level of other employments. [Smith, 1904, 142]

The second feedback mechanism is that the movement of producers into (out of) a sector tends to lower (raise) the payoff through the dual movement of prices.

> If at any time it [the market price] exceeds the effectual demand, some of the component parts of its price must be paid below their natural rate. If it is rent, the interest of the landlords will immediately prompt them to withdraw a part of their land; and if it is wages or profit, the interest of the labourers in the one case, and of their employers in the other, will prompt them to withdraw a part of their labour or stock from this employment. The quantity brought to market will soon be no more than sufficient to supply the effectual demand. All the different parts of its price will rise to their natural rate, and the whole price to its natural price. If, on the contrary, the quantity brought to market should at any time fall short of the effectual demand, some of the component parts of its price must rise above their natural rate. If it is rent, the interest of all other landlords will naturally prompt them to prepare more land for the raising of this commodity; if it is wages or profit, the interest of all other labourers and dealers will soon prompt them to employ more labour and stock in preparing and bringing it to market. The quantity brought thither will soon be sufficient to supply the effectual demand. All the different parts of its price will soon sink to their natural rate, and the whole price to its natural price.[Smith, 1904, 86]

Both of these factors are essential to the process of convergence of market prices to natural prices. Smith understood, however, that convergence is a long-period abstraction, never attained in actual market statistics.

> The natural price, therefore, is, as it were, the central price, to which the prices of all commodities are continually gravitating. Different accidents may sometimes keep them suspended a good deal above it, and sometimes force them down even somewhat below it. But whatever may be the obstacles which hinder them from settling in this centre of repose and continuance, they are constantly tending towards it.[Smith, 1904, 87]

Smith's theory of gravitation implies that observable market prices will have a central tendency, natural prices, as well as fluctuations around this tendency. From a statistical perspective, Smith's theory of prices predicts a *distribution* of prices with a strong modality and a positive, non-infinite dispersion around this modality. His logic of the mechanisms that tend to keep market prices in the neighborhood of natural prices is clearly stated in terms of two negative stabilizing feedbacks. If individual producers didn't pay attention to the expected payoff in deciding where to produce, production would be random and there would be no tendency toward the convergence of market prices to natural prices. If the movement of producers into or out of a sector had no impact on the prices and incomes in the sector through competition, even if producers do seek the highest rates of return, their actions would not lead to a convergence of prices. These considerations indicate that we need to think of the distribution of producers (which implies a distribution of prices and incomes) in terms of an equilibrium joint frequency distribution over the actions of producers and the distribution of producers among sectors of production. We show in the simplest setting that these two parts of Smith's thought experiment imply a Markov process describing the stochastic movement of producers with an ergodic distribution that on average "balances the advantages and disadvantages" of production and implies market prices will gravitate around natural prices.

2 Hub-and-Spoke Model

The problem of organizing the division of labor is expressed as a hub-and-spoke model following [Foley, 2020a]. This model exemplifies the two-fold problem of producers, choosing to diversify or specialize in production and how to distribute themselves among the spokes in order to meet the needs of social reproduction. Smith identifies in *The Wealth of Nations* that producers, who are formally independent from each other, must decide whether to diversify production at the hub and produce all of their needs on their own, or specialize in the production of a particular product at one of the spokes. He identifies that specialization and the division of labor provide a potential for increasing aggregate labor productivity and unlocking the secret to growth and accumulation.

The hub-and-spoke model assumes that there are $N >> 1$ identical independent producers and K spokes, each representing a necessary line of production. The system is described as a vector $\{n_1, n_2, \cdots, n_K\}$, where n_k is the number of producers in spoke k, and $\sum_{k=1}^{K} n_k = N$ is the total number of producers, defining degrees of freedom of the system. Figure 1 illustrates the hub-and-spoke model for 6 different lines of production. This model is well-suited to study how the division of labor is determined by the extent of the market.[2]

The hub-and-spoke model is a useful metaphor for Smith's social coordination problem of achieving the social division of labor through specialized production.[3]

[2] "When the market is very small, no person can have any encouragement to dedicate himself entirely to one employment, for want of the power to exchange all that surplus part of the produce of his own labour, which is over and above his own consumption, for such parts of the produce of other men's labour as he has occasion for." ([Smith, 1904], p.35)

[3] "As it is by treaty, by barter, and by purchase that we obtain from one another the greater part of those mutual good offices which we stand in need of, so it is this same trucking disposition which originally gives occasion to the division of labour. In a tribe of hunters or shepherds a particular person makes bows and

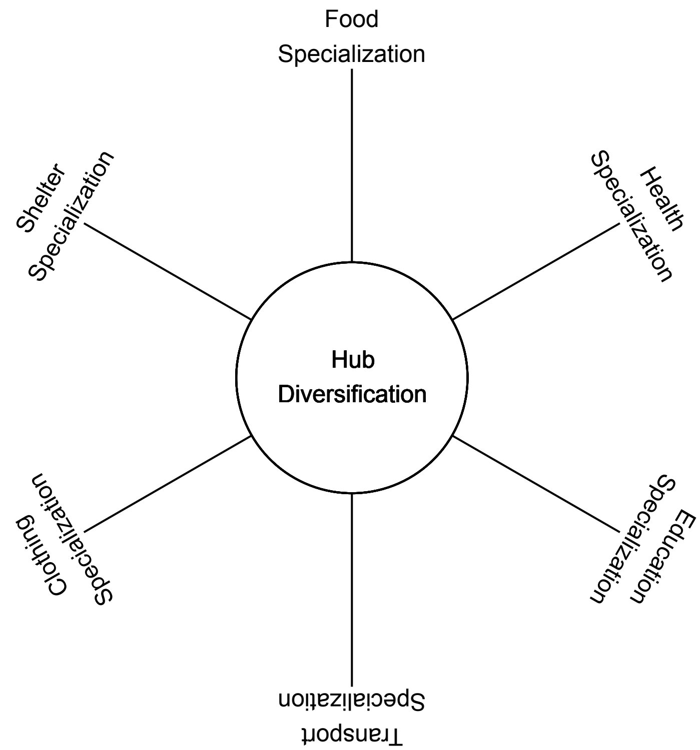

Figure 1: The hub-and-spoke representation of the independent producers decision to diversify and produce all needs for individual reproduction or specialize in the production of a good, or range of goods, that only partially meets their individual needs of reproduction.

2.1 Specialized Production

In the hub-and-spoke model producers' decisions to locate themselves at any particular spoke depends on the shape of the feasible frontier, the relative prices of the goods, and on the context the producers find themselves. We assume that specialized production and exchange is a Cornout-Nash equilibrium and that independent producers face no physical or institutional barriers to exchanging with one another. Thus, producers will tend to find it advantageous to specialize and exchange rather than diversify in production at the hub. An economy of specialized producers sustains a social division of labor.

Agents produce and consume two perishable goods, sugar (s) and corn (c) and need both in a fixed proportion to survive. Producers' payoffs are described by the Leontief function

arrows, for example, with more readiness and dexterity than any other. He frequently exchanges them for cattle or for venison with his companions; and he finds at last that he can in this manner get more cattle and venison than if he himself went to the field to catch them. From a regard to his own interest, therefore, the making of bows and arrows grows to be his chief business, and he becomes a sort of armourer. Another excels in making the frames and covers of their little huts or movable houses. He is accustomed to be of use in this way to his neighbours, who reward him in the same manner with cattle and with venison, till at last he finds it his." ([Smith, 1904], p.31) in the form min$[c, s]$. Because producers specialize in the production of one good they must exchange their surplus for other goods at an exchange ratio. We can choose the units in which producers require each good as being a proportional 1 : 1. The Leontief expression of proportional needs reflects the exogenous social needs of reproduction at a given level of development.

If we take sugar as the money commodity, the price of corn in terms of sugar is $p = \frac{p_c}{p_s}$. This ratio defines a linear price-exchange surface on which a specialized producer will be able to move from a boundary solution to an interior optimum. If labor is indivisible then each producer much decide whether to allocate their labor time to corn or sugar production, the production possibility frontier will consist of three points, two boundary points representing production at each spoke and an origin point representing the zero production. A price system will define payoffs at each spoke as in Figure 2

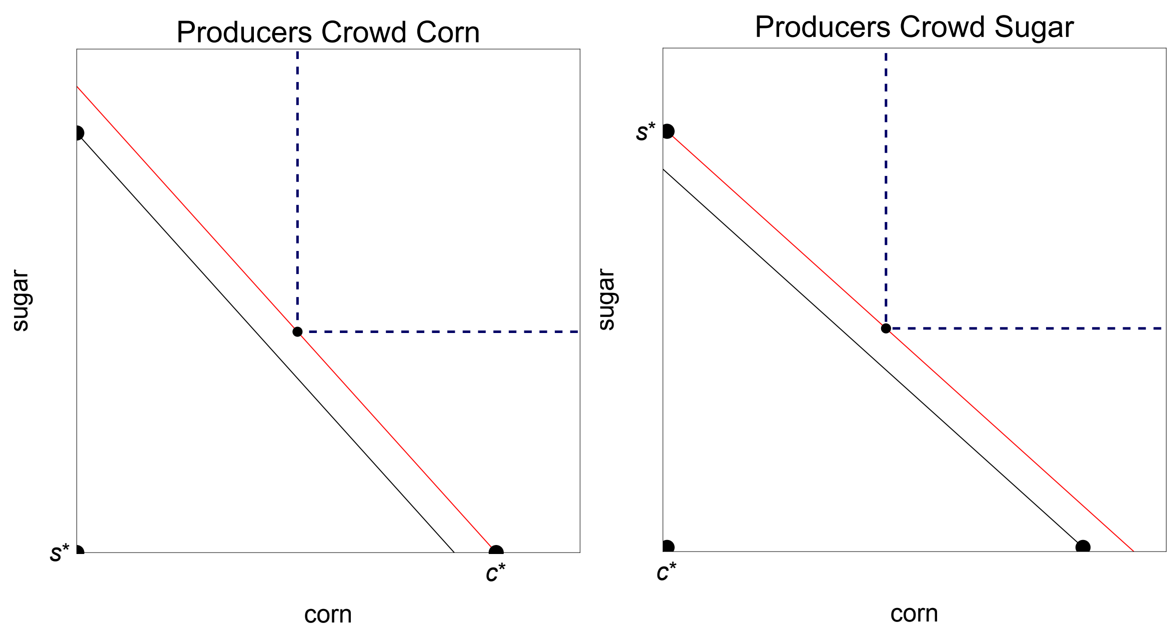

Figure 2: Two price systems, one in which it is advantageous to produce corn (left) and one in which it is advantageous to produce sugar (right). When payoffs are higher for corn production there is a high probability that sugar producers will move to corn production. The opposite will happen when there is an advantage to sugar production. The origin represents the fallback position of the producers.

If prices are such that there is a relative advantage for producing corn, producers who specialize in sugar will begin to exit sugar and crowd corn production. As producers crowd corn production the price of corn and payoff for producing corn will fall. As producers move away from sugar production the price of sugar and payoff for producing sugar will increase. If, for example, the new prices are such that the expected payoff from sugar production becomes higher than corn the typical producer will then exit corn and enter sugar production. These two cases are illustrated in the left and right panels of Figure 2. Smith acknowledges that this migration of labor is a interminable process with no tendency to settle down to a steady-state equilibrium.

The movement of producers, however, does results in the formation of a center of gravity that Smith identifies with natural prices, represented by the line connecting the two specialization corner solutions. At natural prices commodities will exchange at their labor values, $p \propto \frac{\lambda_c}{\lambda_s}$. According to [Smith, 1904, 74],

> [w]hen the price of any commodity is neither more nor less than what is sufficient to pay the rent of the land, the wages of the labour, and the profits of the stock employed in raising, preparing, and bringing it to market, according to their natural rates, the commodity is then sold for what may be called its natural price.

Thus at any one moment we should expect that market prices will deviate from natural prices, but from the continuous migration of labor over and undershooting the relative advantage of employment, a center of gravity emerges around competitive natural prices.

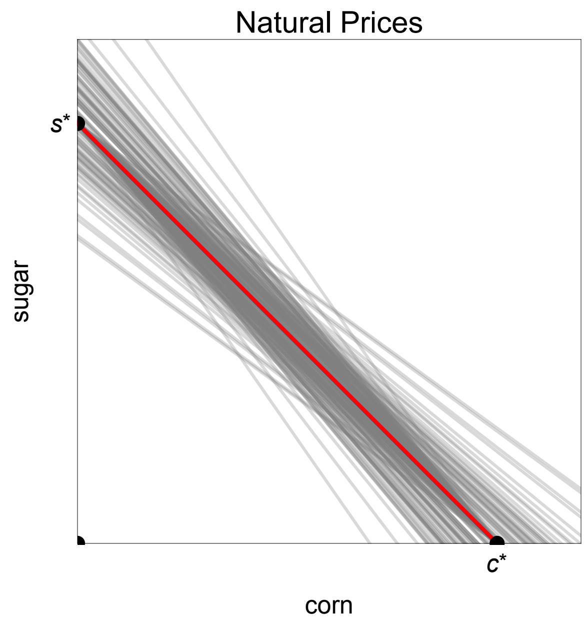

Figure 3: In the natural prices emerge as a center of gravity of market prices. At natural prices the "advantages and disadvantages" are balanced and the number of producers is proportional to labor time in production. The concept of natural prices is illustrated by the red line connecting the two spokes of the PPF. The gray lines illustrate the fluctuations of market prices around the natural price. At natural prices, the payoffs are equalized and producers will have an equal payoff and probability of producing corn or sugar.

An important component of Smith's theory of "gravitational" equilibrium is that even if the incentives are balanced across all spokes an unbalanced allocation of producers will still persist due to the decentralized decision making of independent producers. No individual producer controls the distribution of labor among the different spokes or the production decisions of other producers in a given spoke. No producer knows how much of their commodity is needed by society or how much other producers in their spoke are producing. Thus, the quantity of social labor expended in production is always above or below the effectual demand. Smith is clear in his argument that the balanced staffing over spokes will occur only on average over many cycles of production. Social reproduction in a system of specialized producers can only exist because each disturbance of prices above or below the natural price produces a negative feedback that stabilizes the distribution of prices.

Because the predominant disadvantage of an employment is the labor time required to conduct it, the process of equalization of incomes implies natural competitive prices of commodities will be proportional to their "embodied labor time". Smith's labor theory of value is both an account of the formation of natural prices as well as an account of the distribution of social labor.

In the next section we demonstrate that using methods from statistical physics we can re-conceptualize the spontaneous formation of the division of labor in terms of a statistical equilibrium.

3 Division of labor with two perishable goods

We consider a large number of producers, $N >> 1$, categorized into two distinct groups based on their production: those producing corn, represented as N_c, and those producing sugar, represented as N_s. During a given production period, it is assumed that the total quantity of corn (X) and sugar (Y) produced is equivalent to the number of producers in each respective category. Consequently, the per producer production of corn (n_c) and sugar (n_s) can be calculated as $n_c = \frac{X}{N}$ and $n_s = \frac{Y}{N} = 1 - n_c$, respectively. We follow Smith and assume that if any intermediate goods are necessary for production they will emerge in terms of the final good and do not assume the social relations of capital.

As noted by [Bowles, 2022], in markets that do not clear, there are two sides: the "long-side" and the "short-side". On the "long-side", either supply or demand, has a greater number of desired transactions at a given price. However, not all participants on this side will be able to complete their desired transactions, leading to some being quantity constrained. On the other hand, the "short-side", which can also be either supply or demand, has fewer desired transactions. Participants on the "short side" are able to complete all their desired transactions and retain a "short-side" power over the "long side". [4]

We assume that after a production period the prices for a good produced on the "long side" of the market will become so small that the producers on the short side will effectively get the entire surplus from market exchange.

In the hub-and-spoke model an excess supply of corn implies $n_c > \frac{1}{2}$ in which case corn will be on the "long side" and sugar on the "short side". Symmetrically, if there is excess supply of sugar and $n_c < \frac{1}{2}$, sugar will be on the "long side" and corn the "short side".

If at the end of a production period there is excess supply of sugar because there are too many producers in sugar, then $n_c < \frac{1}{2}$ and corn producers are on the "short-side" and sugar producers are on the "long-side" of the market. In this case the price of sugar will become so small that a sugar producer will get zero corn and zero sugar for payoff $\min[0, 0] = 0$. Corn producers on the "short side" will get the $\frac{X}{N_c} = 1$ unit of corn they produce as well

[4] Producers on the short-side side "Where characteristics of an exchange preclude contracting for specific agent services, and where there is a potentially long-term relationship between the exchanging parties, the threat of non-renewal of the exchange by one of the parties may serve as a sanctioning device. In such cases markets will not clear in competitive equilibrium, and agents on the "short-side" of the market hold power over agents on the "long-side" of the market. We term this short-side power."[Bowles and Gintis, 1998]

as some fraction of the sugar produced. If we assume that each corn producer gets an equal average amount of sugar they will then effectively divide all produced sugar equally between corn producers, $\frac{Y}{N_c} = \frac{n_s}{1-n_s} = \frac{1-n_c}{n_c}$. In this case, a typical corn producer's payoff is $\min[1, \frac{1-n_c}{n_c}] = 1$. Equivalently, we can say that the "typical" producer will have a payoff $\min[0,0] = 0$ with probability $1 - n_c$ and a payoff $\min[1, \frac{1-n_c}{n_c}] = 1$ with probability n_c. Schematically, the typical producer faces the following payoffs:

$$n_c < \frac{1}{2} \begin{cases} \text{With Prob.} = n_c \begin{cases} \text{Corn: } \frac{X}{N_c} = 1 \\ \text{Sugar: } \frac{Y}{N_c} = \frac{n_s}{1-n_s} = \frac{1-n_c}{n_c} \end{cases} & \to \min\left[1, \frac{1-n_c}{n_c}\right] = 1 \\ \\ \text{With Prob.} = 1 - n_c \begin{cases} \text{Corn: } 0 \\ \text{Sugar: } 0 \end{cases} & \to \min[0,0] = 0 \end{cases} \tag{1}$$

When sugar is on the "long side", there will be an excess of sugar equal to $1 - 2n_c > 0$ that will be disposed of at the end of the period. No accumulation is possible when both goods are perishable.

By symmetry of the goods, when sugar is on the "short-side" and $n_c > \frac{1}{2}$, assuming equal distribution of corn among sugar producers the typical producer will face the following payoffs:

$$n_c > \frac{1}{2} \begin{cases} \text{With Prob.} = 1 - n_c \begin{cases} \text{Corn: } \frac{X}{N_s} = \frac{n_c}{1-n_c} \\ \text{Sugar: } \frac{Y}{N_s} = 1 \end{cases} & \to \min\left[\frac{n_c}{1-n_c}, 1\right] = 1 \\ \\ \text{With Prob.} = n_c \begin{cases} \text{Corn: } 0 \\ \text{Sugar: } 0 \end{cases} & \to \min[0,0] = 0 \end{cases} \tag{2}$$

In Smith's system, direct commodity producers respond to payoff differences by moving from disadvantageous lines of production to advantageous lines of production. The decentralized nature of decision making, however, implies the typical producer will move from one line of production to another probabilistically. A simple and parsimonious way of modeling the partial randomization of strategies is by constraining the typical producer's mixed strategy with a minimum informational entropy [Foley, 2020b; Scharfenaker, 2020]. In the hub-and-spoke model producers much choose to produce at one of the K spokes representing the production of specific commodities. We can represent producers' choice problem as choosing an action $\{a_1, \cdots, a_K\}$ knowing the payoff $u(a_k)$ associated with each action.

We can represent the typical producer's mixed strategy in terms of a frequency distribution $\{f_1, \cdots, f_K\}$, $\sum_k f_k = 1$, and expected payoff $\sum_k f_k u(a_k)$. Maximizing the expected payoff subject to only the normalization of frequencies implies that the producer will choose the line of production with the highest payoff with a probability of unity. Such a deterministic description of producers' actions is theoretically incompatible with Smith's thought experiment as it implies zero fluctuations in the distribution of producers and no social division of labor. If we constrain producers' mixed strategy with a minimum entropy the maximization problem is

$$\begin{aligned} \max_{\{f \geq 0\}} \sum_k f_k u(a_k) \\ \text{subject to} \quad \sum_k f_k = 1 \\ -\sum_k f_k \log(f_k) \geq H_{\min} \end{aligned} \tag{3}$$

The Lagrangian associated with this maximization problem, with the two Lagrangian multipliers T and μ is:

$$\begin{aligned} L = -\sum_k f_k u(a_k) - \mu \left(\sum_k f_k - 1 \right) \\ +T \left(-\sum_k f_k \log(f_k) - H_{\min} \right) \end{aligned} \tag{4}$$

The Lagrange multiplier μ ensures the normalization of the frequencies over actions. We refer to T as the "behavior temperature" since it measures the scale of fluctuations of individual behavior resulting from the sensitivity of agents to differences in payoffs. A lower T makes agents more sensitive to differences in payoffs as the constraint is less binding. The solution to the constrained maximization problem is the Gibbs (SoftMax) distribution over actions.

$$f(a_k) = \frac{e^{\frac{u[a_k]}{T}}}{\sum_k e^{\frac{u[a_k]}{T}}} \tag{5}$$

When there are just two actions, such as $a_1 =$ corn and $a_2 =$ sugar, the solution reduces to:

$$f(a_c) = \frac{e^{\frac{u(a_c)}{T}}}{e^{\frac{u(a_c)}{T}} + e^{\frac{u(a_s)}{T}}} = \frac{1}{1 + e^{\frac{u(a_s) - u(a_c)}{T}}} \tag{6}$$

$$f(a_s) = 1 - f(a_c) = \frac{1}{1 + e^{-\frac{u(a_s)-u(a_c)}{T}}} \tag{7}$$

The Gibbs distribution implies producers will choose each line of production with a positive frequency. For two actions, such as corn and sugar, the relative logs odds of choosing an action is just the difference in payoffs scaled by T:

$$\log\left(\frac{f_s}{f_c}\right) = \frac{u(a_s) - u(a_c)}{T} \tag{8}$$

If corn producers' expected payoff for producing in period $t+1$ is just their payoff from producing at time t, they will move from one line of production another with probability:

$$f\left(n_c(t+1)|n_c(t)\right) = \begin{cases} \frac{1}{1+e^{\frac{Min[0,0]-Min\left[1,\frac{1-n_c(t)}{n_c(t)}\right]}{T}}} = \frac{1}{1+e^{-\frac{1}{T}}} & \text{if } n_c(t) < \frac{1}{2} \\ \frac{1}{1+e^{-\frac{Min\left[1,\frac{1-n_c(t)}{n_c(t)}\right]-Min(0,0)}{T}}} = \frac{1}{1+e^{\frac{1}{T}}} & \text{if } n_c(t) > \frac{1}{2} \end{cases} \tag{9}$$

We can visualize the expected number of producers in each line of production as a function of the temperature T. When $T \to 0$ producers will be entirely crowded into one line of production. As $T \to \infty$ producers become distributed uniformly between sugar and corn production due to the weakness of the feedback of payoffs on production decisions. It is important to recognize that the limiting "degenerate" case in which $T \to 0$ and all endogenous fluctuations abate cannot support a division of labor or Smith's theory of value. The purposive decentralized nature of decision making in unplanned market economies is not likely to approximate the deterministic "degenerate" case in which any infinitesimal deviation in payoffs results in the instantaneous wholesale migration of labor. We should expect that producers operate between these two extremes, where payoffs incentivize producers to move, but the decentralized market interactions make this movement probabilistic.

The stochastic quantal response of the typical producer induces a Markov chain on the state space of profiles of agent behavior. If there are N producers each with the same behavioral temperature T, the state of the system (distribution of producers) is described by the number of producers choosing corn, $N_c = 0, 1, ..., N$. The frequency with which each producer will choose corn is $f(n_c) = \frac{1}{1+e^{-\frac{1}{T}}}$ and takes the Binomial form:

$$\binom{N}{1 - N_c} f(n_c)^{1-N_c}(1 - f(n_c))^{N-(1-N_c)} \tag{10}$$

Because the Markov transition matrix is an $N \times N$ irreducible, non-negative row-stochastic matrix, the Perron-Frobenius theorem tells us that the normalized eigenvector associated with the largest eigenvalue (equal to one) is the ergodic distribution of the Markov chain,

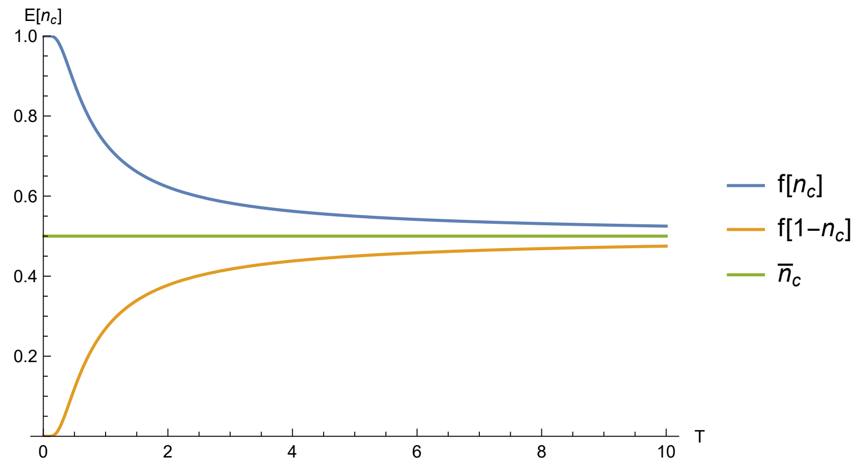

Figure 4: The expected number of producers fluctuate from all crowded into one line of production for temperature equal to zero and bifurcate as temperature goes to infinity.

that is the stationary distribution of producers as the number of production periods becomes large.

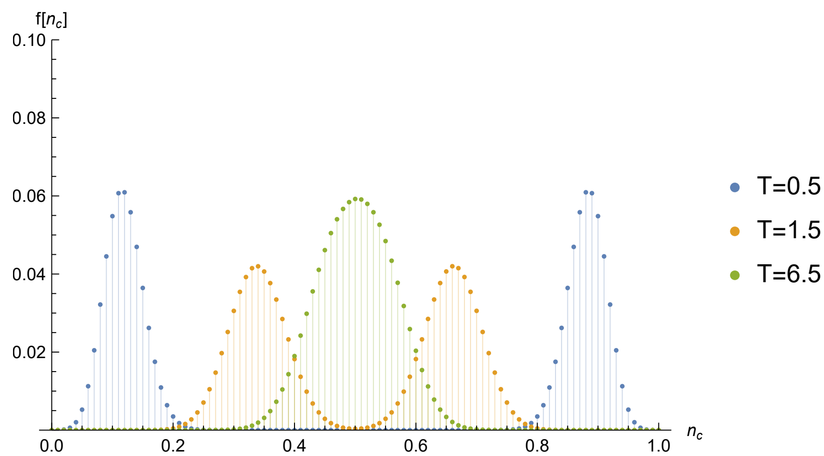

Figure 5: The ergodic distribution for division of labor with two perishable goods in the form of a binomial distribution for different behavioral temperatures.

Figure 5 represents the ergodic distribution of 100 producers (normalized) for different values of T. When T is small producers will tend to crowd into one line of production, as is seen in the bimodal distribution. As T becomes larger, producers will tend to fluctuate more between the two spokes. In all case the expected number of producers, $E(n_c)$ will correspond to Smith's balanced "advantages and disadvantages" of employment and labor theory of value.

3.1 Inequality

While the perishable goods economy is only meant as a benchmark, it does imply production and exchange will endogenously generate inequality.

Figure 6 shows the Gini index and a Lorenz curve to present the results in terms of inequality in this model for 100 producers, showing the population of corn producers in the x-axis.

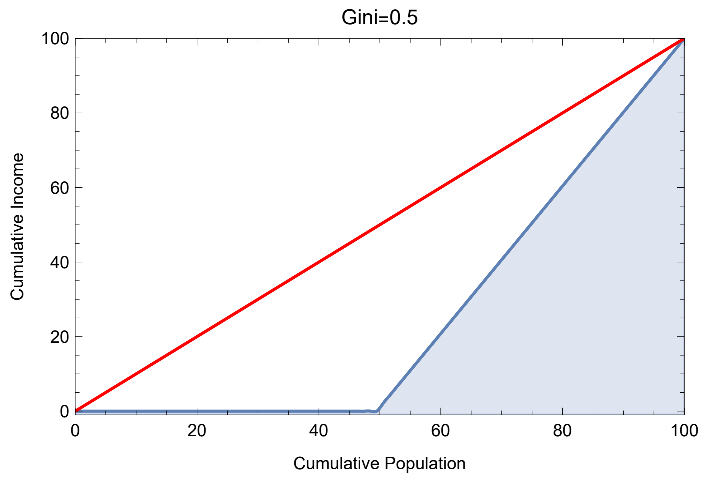

Figure 6: The Lorenz curve for the perishable goods economy at the ergodic distribution of producers. The Gini coefficient is equal to 0.5.

The blue line is the Lorenz curve corresponding to the cumulative percentage of the population and the cumulative percentage of goods owned by that portion of the population. The red line is the line of perfect equality, which would represent a situation where each person has the same share of goods. The Gini coefficient, ratio of the area between the Lorenz curve and 45−degree line to the total area under the 45−degree line is 0.5.

4 Discussion

Smith's "gravitational" equilibrium theory of natural prices has been predominantly articulated within a "long-period" framework that acknowledges the endogenous fluctuations of market prices, but because of "how intrinsically complex the issue of gravitation is"[Kurz and Salvadori, 1995] finds a tractable point of departure in abstracting away from these statistical complexities. One unfortunate effect of the success of the "long-period" approach was that it turned attention away from the development of a coherent statistical methodology in classical political economy. We emphasize that statistical equilibrium is not an approximation to idealized systems operating at zero entropy. The conclusions one might draw from the "stylized" economy are often not supported in statistical equilibrium [Scharfenaker and Foley, 2023]. In this paper we demonstrate how a division of labor can spontaneously self-organize and be sustained over time due to the fluctuations of producers across lines of production. Producers behave purposively in their decisions seeking out the highest rate of remuneration. The decentralized nature of market interactions induces a minimum entropy on producers' action set leading to a non-degenerate equilibrium distribution of producers across all lines of production.

Non-degenerate prices and incomes in equilibrium tends to be understood in terms of a system in disequilibrium. The movement of agents in statistical equilibrium, however, is conceptually distinct from the study of disequilibrium dynamics (such as those studied in Duménil and Lévy [1991]). A statistical equilibrium model substitutes a probabilistic description of the system, in terms of the configurations of the system, for a detailed dynamic prediction of the movement of each individual part. A system in statistical equilibrium is defined by a frequency distribution over all states of the system. In the hub-and-spoke framework this implies an equilibrium frequency distribution of producers and income when decentralized producers respond to payoff differences and can accumulate the money commodity.

As [Kurz and Salvadori, 1995, pp.20] acknowledge, "A proper answer to it [how to model gravitational equilibrium] would seem to contain, of necessity, an answer to many economic questions which are as yet unresolved." We believe that the statistical equilibrium approach to Smith's gravitational equilibrium proves a step in the right direction to answering this question. Articulating a theory of the statistical effects of market mediated production and exchange in systems with many agents interacting in complex ways is fundamental to revealing and understanding the statistical regularities in economic data.

5 Conclusion

Classical Political Economy recognized capitalism as a complex social system. The astronomical degrees of freedom, complex interdependencies, interactions, and numerous feedbacks make modeling complex systems a formidable task. Because of the complexity of the system classical political economists such as Adam Smith argued that any conclusions drawn about capitalism must rest on robust, pervasive, self-reinforcing (statistical) tendencies. The emergence of natural prices from the free mobility of independent producers is a simple yet powerful illustration of Smith's methodological approach. Smith's logic concerning the process of the spontaneous formation of the social division of labor and its implications for the theory of value is inherently statistical. Centers of gravity in prices emerge through the endogenous fluctuations of individual producers between different lines of production. Free competition and the decentralized nature of production decisions implies the movement of producers among different lines of production is stochastic. We address this irreducible element of randomness in Smith's theory by developing a statistical equilibrium hub-and-spoke model whereby entropy constrained independent producers balance the "advantages and disadvantages" of employment through their stochastic movement between spokes. The resulting ergodic distribution of producers supports Smith's theory of gravitational equilibrium and the labor theory of value.

Statistical equilibrium methods shed new light on the primary theoretical abstractions of Classical Political Economy by modeling the statistical processes that generate the robust predicted regularities.

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